Stochastic process model for damage
Damage modelling

Jim Zidek

February 21, 2017
In general loads on wooden structures are classified as dead (e.g. weight of the roof) or live (e.g. snow on the roof).

A dead load imposes a constant load that leads to “creep” and possible “creep failure” for this visco–elastic material, the duration of load (DOL) effect.

A live load imposes a short term load that if cause failure from a different mechanism, the failure load being dependent on the rate of loading (ROL) effect. ROL and DOL are believed to be related.

Design values must incorporate a safety factor to account for the dead and live loading effects.
Stochastic damage model for live loads:

Let \( \{ Y_t \} \) represent increasing stochastic damage process that for times \( t = t_j \) and \( t = t_{j-i} \) is

\[
Y_{t_j} - Y_{t_{j-1}} = \sum_{k=1}^{N_{(t_{j-1}, t_j)}} X_k,
\]

where

- \( N_{(t_{j-1}, t_j)} \sim Poisson(\Lambda_{(t_{j-1}, t_j)}) \) and
- \( \Lambda_{(t_{j-1}, t_j)} = \int_{t_{j-1}}^{t_j} \lambda_x \, dx. \)
NOTES:

1: So for each \( k \), \( E(e^{iuX_k}) = (1 - iu\xi)^{-\eta} \).

- For fixed \( N_{(t_{j-1}, t_j)} = m \), \( Y_{t_j} - Y_{t_{j-1}} \) also has a gamma distribution.
- Also each \( X_k \) is infinitely divisible i.e. for any \( n \):

\[
E(e^{iuX}) = \left[(1 - iu\xi)^{-\left(\frac{\eta}{n}\right)}\right]^n
\]

Implies **Levy–Khinchine formula** must hold:

\[
- \log(e^{iuX_k}) = \eta \log \{(1 - iu\xi)\} = \eta \int_{0}^{\infty} (1 - e^{iux})G(dx)
\]

with \( G(dx) = \exp (-x/\xi) / x \ dx \).
NOTES:

2:  \( E[Y_{t_i} - Y_{t_{i-1}}] = \xi \eta \Lambda(t_{i-1}, t_i) \)

3:  \( \text{Var}[Y_{t_j} - Y_{t_{j-1}}] = \xi \eta \Lambda(t_{j-1}, t_j)[\eta + \xi] \)

Example: Snow loads (Foschi)
Modelling creep: and infinite sequence of infinitesimal jumps

Let intensity $\lambda \to \infty$ and $\xi \to 0$.

**Result:** Gamma (Levy) process:

$$Y_t \sim \text{Gamma}(\eta_t, \xi)$$

where $x_i = \xi(x)/Z$ involves a random effect $z$ and covariates $x$ such as strength reducing characteristics. One choice (Lawless) for $Z \sim \text{Gamma}(\gamma^{-1}, \delta)$ offers tractibility.