Statistical framework for DOL models and degradation: a review and current progress

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Lumber is a very popular construction material: must consider its long-term reliability and load-carrying capacity in the design of wood-based structures.

Application of structural reliability concepts to wood products is challenging: lumber has considerable inherent variability.

The strength of wood changes over time due to applied stresses as a function of:

- duration of the load – known as DOL effect (Wood, 1951)
- rate at which the load is applied – known as ROL effect (Liska, 1950)
Background

- Impractical to perform experiments that measure time to failure of wood products with a long intended life span (e.g. 50 or more years)
- Experiments based on accelerated testing have been carried out on lumber specimens to collect empirical data for assessing the impact of DOL and ROL effects and quantifying the loss of load-carrying capacity
- To predict time-to-failure under stochastic loadings when lumber is placed in service, models were needed
Recap: Progress to date

Paper #1: Dimensional and statistical foundations for accumulated damage models

Authors: Samuel Wong and Jim Zidek
Status: Published in *Wood Science and Technology* (2018), 52(1), 45-65.
Summary. Lays foundations for nondimensionalizable accumulated damage models (ADMs) e.g. US and Canadian models. Demonstrates application of these models with a Bayesian analysis of ramp load data.
Recap: Progress to date

Paper #2: Bayesian analysis of accumulated damage models in lumber reliability

Authors: Chun-Hao Yang, Jim Zidek, Samuel Wong
Status: Published in Technometrics (2019), 61(2), 233-245.
Summary. Develops statistical platform for fitting ADMs using ABC, in particular for quantifying uncertainties in the reliability of lumber under the DOL effect. Compares revised Canadian model with original.
Recap: Progress to date

Paper #3: The duration of load effect in lumber as stochastic degradation

Authors: Samuel Wong and Jim Zidek
Status: Published in *IEEE Transactions on Reliability* (2019), 68(2), 410-419.
Summary. Proposes a distinct approach from ADMs, using a gamma process to model damage as is common in degradation modeling applications. Advantages include having parameters that are easier to interpret and simpler computation.
Recap: Progress to date

Paper #4: Calibrating wood products for load duration and rate: A statistical look at three damage models

**Authors:** Samuel Wong  
**Status:** Submitted to *Wood Science and Technology*  
**Summary.** Reviews the US, Canadian, and gamma process models for damage. Proposes extensions to fit data from 3 types of load profiles and quantify uncertainty. Compares and applies models on the complete Hemlock dataset generated by Forintek in the 1980s.
A renaissance in DOL research?

With the introduction of new engineered wood products such as oriented strand board and cross-laminated timber, DOL is a continued avenue of research.

Other groups publishing recently include


Accelerated testing: Experimental setup in Forintek laboratory in Richmond, BC

From Foschi and Barrett (1982)
Accelerated testing: Three loading scenarios used in calibration experiments

(a) Ramp load
(b) Ramp-Constant load
(c) Ramp-Constant-Ramp load

Adapted from Wang and Yang (2019)
Table: Summary of ramp load data from the Forintek Canada experiment. Five different rates of loading were used, as indicated in the Rate column in terms of $k_s$.

<table>
<thead>
<tr>
<th>Group</th>
<th>Rate $k \times k_s$</th>
<th>sample size</th>
<th>mean time-to-failure</th>
<th>mean MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.667 \times 10^{-3}$</td>
<td>140</td>
<td>619 minutes</td>
<td>44.80</td>
</tr>
<tr>
<td>2</td>
<td>0.0333</td>
<td>139</td>
<td>31.8 minutes</td>
<td>46.35</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>139</td>
<td>65.7 seconds</td>
<td>47.83</td>
</tr>
<tr>
<td>4</td>
<td>30.0</td>
<td>139</td>
<td>2.13 seconds</td>
<td>46.59</td>
</tr>
<tr>
<td>5</td>
<td>1500</td>
<td>140</td>
<td>0.0453 seconds</td>
<td>48.95</td>
</tr>
</tbody>
</table>

$k_s$ is rate used to define ‘short-term strength’, about a 1-min test.
Table: Summary of constant load and RCR data from the Forintek Canada experiment. Five different combinations of constant load level and duration were used, as indicated in the $\tau_c$ and $T_1$ columns respectively.

<table>
<thead>
<tr>
<th>Group</th>
<th>$\tau_c$</th>
<th>$T_1$</th>
<th>#samples</th>
<th># failures during time interval</th>
<th># failures during time interval</th>
<th># failures during time interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>20.68</td>
<td>3 mo.</td>
<td>300</td>
<td>17</td>
<td>18</td>
<td>265</td>
</tr>
<tr>
<td>7</td>
<td>20.68</td>
<td>4 yr.</td>
<td>198</td>
<td>4</td>
<td>42</td>
<td>152</td>
</tr>
<tr>
<td>8</td>
<td>31.02</td>
<td>3 mo.</td>
<td>98</td>
<td>19</td>
<td>26</td>
<td>53</td>
</tr>
<tr>
<td>9</td>
<td>31.02</td>
<td>1 yr.</td>
<td>300</td>
<td>57</td>
<td>97</td>
<td>146</td>
</tr>
<tr>
<td>10</td>
<td>31.02</td>
<td>4 yr.</td>
<td>101</td>
<td>23</td>
<td>41</td>
<td>37</td>
</tr>
</tbody>
</table>

▶ Full dataset not previously analyzed using damage models
Models for DOL: damage accumulation approach

- \( \tau(t) \): load applied as a function of time
- \( \alpha(t) \): damage state of the piece as a function of time
- \( \alpha = 0 \) indicates no damage, and \( \alpha = 1 \) indicates failure
- Accumulated damage models (ADMs) developed for \( \alpha(t) \), to fit accelerated test data with known \( \tau(t) \)
- **Key idea**: use fitted model to estimate reliability under arbitrary \( \tau(t) \).
Models for DOL: The US model

Expressed as an ODE:

\[
\frac{d}{dt} \alpha(t) \mu = \exp \left( -A + B \frac{\tau(t)}{\tau_s} \right)
\]

where

- \(0 \leq \alpha(t) \leq 1\) is the damage accumulated by time \(t\)
- \(\alpha(T_f) = 1\) indicates failure occurs at the (random) time \(T_f\)
- \(\mu\) constant with unit ‘time’
- \(\tau_s\) (psi) = short term breaking strength, assumed log-normal: \(\tau_s \sim \tau_M \exp(wZ)\), set \(\tau_M\) to be median short-term strength in data
- Parameters to estimate are \(A, B, w\)
- NLS estimation procedure due to Gerhards (1987)
Models for DOL: The Canadian model

Expressed as an ODE:

\[
\frac{d}{dt} \alpha(t) \mu = \left[ (a\tau_s)(\tau(t)/\tau_s - \sigma_0)_+ \right]^b + \left[ (c\tau_s)(\tau(t)/\tau_s - \sigma_0)_+ \right]^n \alpha(t)
\]

where

- \(0 \leq \alpha(t) \leq 1\) is the damage accumulated by time \(t\)
- \(\alpha(T_f) = 1\) indicates failure occurs at the (random) time \(T_f\)
- \(\tau_s\) (psi) = short term breaking strength
- \(\mu\) constant with unit ‘time’
- \(a, b, c, n, \sigma_0\) are piece-specific random effects that provide randomness in \(T_f\), effects assumed to be log-normal
- \(\sigma_0\) is the stress ratio threshold: damage starts to accumulate only when \(\frac{\tau(t)}{\tau_s} > \sigma_0\).
Models for DOL: The gamma process model

- Stochastic process representing damage: $Y(t) \geq 0$
- $Y(T_f) = 1$ indicates failure occurs at the (random) time $T_f$
- Damage accumulated from time $t_1$ to $t_2$, i.e., $Y(t_2) - Y(t_1)$, has a gamma distribution with scale parameter $\xi$ and shape parameter $\eta(t_2) - \eta(t_1)$, where $\eta(t)$ non-decreasing
- For constant load $\tau$ from time 0 to $t$, a model for the shape parameter is

$$\eta(t) = u g(t) \times (\tau - \tau^*)_+$$

- $\tau^*$: load threshold
- $g(\cdot)$: an increasing function for DOL effect
- $u$: scaling parameter
Models for DOL: The gamma process model

- Generalize to arbitrary loadings:
  \[
  \eta(t) = u \sum_{i=1}^{m} g(\tilde{t}_i) \times [(\tau_i - \tau^*)_+ - (\tau_{i-1} - \tau^*)_+] 
  \]

- \(0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_m\): sequence of load levels
- \(\tilde{t}_i\): the total time duration for which \(\tau(t) \geq \tau_i\)
- \(g(\tilde{t}_i)\): the DOL effect of the load increment from \(\tau_{i-1}\) to \(\tau_i\)

- \(\eta(t)\) is deterministic given \(\tau(t)\), thus variability between pieces naturally captured by \(Y\)

- Adopt broken power-law form: \(g(t) \propto (t/t_i)^{a_i}\) for \(t_{i-1} < t \leq t_i\), where \(t_0 = 0\) and \(t_1, t_2, \ldots\) is a sequence of time breakpoints with powers \(a_1, a_2, \ldots\) to be estimated.

- Closed-form likelihood function (unlike ADMs)
Reliability assessment workflow

1. Perform accelerated testing experiments
   ▶ ramp-load, constant-load, and RCR tests

2. Estimate ADM parameters
   ▶ Previously difficult to assess uncertainty in estimates: use new principled statistical methods

3. Generate stochastic loadings $\tau(t)$
   ▶ simulate real conditions such as snow loads and occupancy loads over 50+ years

4. Use fitted ADM to compute reliability indices and develop safety factors.
   ▶ Need to coherently propagate uncertainty from parameter estimation to reliability estimation
Figure: QQ plots for the fitted US model.
Figure: QQ plots for the fitted Canadian model.
Figure: QQ plots for the fitted gamma process model.

Estimated BICs for US, Canadian, and gamma process models:
-5898, -6188, and -6184
Reliability application

Structures encounter different live load patterns, e.g., owner occupancy in residential units, office occupancy in commercial buildings, and snow loads on roofs.

Example residential loads model for $\tau(t)$, $t \geq 0$:

$$
\tau(t) = \phi R_o \frac{\gamma \tilde{D}_d + \tilde{D}_s(t) + \tilde{D}_e(t)}{\gamma \alpha_d + \alpha_l}
$$

Simulate a large number (100,000) of these profiles to estimate probability of failure $\hat{p}_f$ over 50 year period.

Convert to z-score, $\hat{\beta} = -\Phi^{-1}(\hat{p}_f)$.

Repeat for different performance factors $\phi$. 
Figure: An example residential load profile.
Reliability application
Summary

- With thanks to Conroy and Erol for introducing us to this important area of research and giving us extensive advice along the way, we have built a solid statistical framework for future DOL researchers.
- Account for estimation uncertainty within each model.
- Computation now possible with multiple models, which is a good thing since all models are approximations!
- Code is available in a public Github repository.