Our revised US model:

\[
\frac{d\alpha(t)}{dt} \mu_f = \exp \{ -A + B \frac{\tau(t)}{\tau_s} \}.
\]

where

- \(0 \leq \alpha(t) \leq 1\) is the damage accumulated by time \(t\)
- \(\alpha(T_f) = 1\) indicates that failure occurs at time \(T_f\)
- \(\tau_s\) (psi) = short term breaking strength
- \(\mu_f\) population mean time to failure under load \(\tau(t)\)
- \(A, B\) unitless parameters
Implications of US model

After integrating the differential equation and some manipulations, we get

\[ \alpha(t) = \frac{\int_0^{(t/\mu_f)} \exp \left\{ B \frac{T(u \mu_f)}{\tau_s} \right\} du}{\int_0^{(T_f/\mu_f)} \exp \left\{ B \frac{u \mu_f}{\tau_s} \right\} du}. \]  

(1)

Note that ODE implies that \( T_f, A, B \) are dependent, in the sense that knowing two of those quantities determines the third.

In this case we have eliminated the parameter A.
Consider the “ramp-load” pattern, \( \tau(t) = kt \), where \( k \) is the loading rate.

In fact, the short-term strength \( \tau_s \) is often taken to be the strength under the ramp-load test, so that \( \tau_s = kT_s \).

Under these conditions, we can simplify to

\[
\alpha(t) = \frac{\exp\{Bt/T_s\} - 1}{\exp\{B\} - 1}
\]

The implications of current DOL models are damage-accumulation functions that are not observable except at \( \alpha(0) = 0, \alpha(T_s) = 1 \). Instead it provides a framework on which to hang the various elements of the model.
Revised Canadian model

\[ \pi_1(t) = [(\tilde{a}\tau_s)(\pi_3(t) - \sigma_0)_+]^b + [(\tilde{c}\tau_s)(\pi_3(t) - \sigma_0)_+]^n\alpha(t) \]

- \(\tilde{a}\) and \(\tilde{c}\) are now random effects with \([\tilde{a}] = [\tilde{c}] = F^{-1}L^2\)
- Both sides of the equation are now unitless

Under ramp-load this becomes:

\[ \dot{\alpha}(t)\mu_f = [\tilde{a}kT_f(t/T_f - \sigma_0)_+]^b + [\tilde{c}kT_f(t/T_f - \sigma_0)_+]^n\alpha(t). \]

where \(k\) is the loading rate.
Most previous work assumes that the parameters in the models are piece-specific random effects, with a Normal or Log-normal distribution, e.g. for the US model

\[ A_i \sim LN(\mu_A, \sigma^2_A), \quad B_i \sim LN(\mu_B, \sigma^2_B) \]

Foschi’s approach is to simulate values \( A_i \) and \( B_i \), calculate \( T_f \), and try to match the empirical CDF of the data.

Principled statistical methods for parameter estimation are needed!
Learning DOL models from data

- We tried a likelihood-based approach for fitting the approximate ramp load data from previous FPI experiments to revised US and Canadian models.
- Very computationally intensive for Canadian model – requires numerically solving $T_f$ for likelihood function calculations.
Assessment of model fits via empirical cumulative distributions of data generated from fitted models.

(a) US Model  (b) Canadian Model